

Images of F

In the *Images of F* applet, the columns of the matrix are the elementary vectors e_1 and e_2 . The blue figure is a pre-image initially in the shape of an F. The green figure is the image of the blue F under the transformation given by the matrix.

To answer the questions below, you can drag the tips of the elementary vector to set up the appropriate matrices. You may also need to drag the vertices of the blue F as well.

Warm Up: Set up the following matrices one at a time. Pay particular attention to the lattice points of F and to the lattice points of the image of F.

(a) $\begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 \\ 3 & -1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix}$ (e) $\begin{pmatrix} -2 & 1 \\ 2 & -1 \end{pmatrix}$ (f) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

Investigation 1: Drag the tips of the elementary vectors to set up the following matrices. Discuss the transformations and the resulting image of F under these matrix transformations.

(a) Transformations with matrices of the form $\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$.

(b) Transformations with matrices of the form $\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$.

(c) Transformations with matrices of the form $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$.

(d) Transformations with matrices of the form $\begin{pmatrix} 0 & k \\ k & 0 \end{pmatrix}$.

(e) Transformations with matrices of the form $\begin{pmatrix} 1 & 0 \\ k & 1 \end{pmatrix}$.

(f) Transformations with matrices of the form $\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$.

Investigation 2: Drag the tips of the elementary vectors to set up matrices that will perform the following transformations. Pay attention to the orientation of the vectors.

- (a) Reflection over the x – axis.
- (b) Reflection over the y – axis.
- (c) 90-degree clockwise rotation around the origin.
- (d) Half-turn around the origin.
- (e) 90-degree counter clockwise rotation around the origin.
- (f) Reflection over the line $y = x$.
- (g) Reflection over the line $y = -x$.